Recursion-9

11:56

Let $f(n) + c_1 f(n-1) + c_2 f(n-2) + \dots + c_k f(n-k) = q(n)$

where

$q(n) = q\,b^n$ where q is const.

For particular soln $f(n) = n^m d\, b^n$ where m is the multiplicity of b as char root.

Q:→ Solve

$$f(n) + 5f(n-1) + 6f(n-2) = 42(4)^n$$

sol:→ Homo soln $f^h(n)$:

Associated Homo eqn $f(n) + 5f(n-1) + 6f(n-2) = 0$

For char eqn, take $f(n) = a^n$

$$\Rightarrow a^2 + 5a + 6 = 0$$

$$\Rightarrow a = -2, -3$$

$$\therefore f^h(n) = A(-2)^n + B(-3)^n$$

Particular soln $f^p(n)$

As $q(n) = 42(4)^n$ where 4 is not char root.

For particular soln, take $f(n) = d(4)^n$ in the given recc. relation

$$f(n) + 5f(n-1) + 6f(n-2) = 42(4)^n$$

$$\Rightarrow d(4)^n + 5d(4)^{n-1} + 6d(4)^{n-2} = 42(4)^n$$

$$\Rightarrow d(4)^2 + 5d(4) + 6d = 42(4)^2$$

$$\Rightarrow 16d + 20d + 6d = 42 \times 16$$

$$\Rightarrow 42d = 42 \times 16$$

$$d = 16$$

$$\therefore f^p(n) = d(4)^n = 16(4)^n = 4^{n+2}$$

Complete soln

$$f(n) = f^h(n) + f^p(n) = A(-2)^n + B(-3)^n + 4^{n+2}$$

Q:→ Solve $f(n) - 3f(n-1) - 4f(n-2) = 4^n$

sol:→ Homo soln ($f^h(n)$)

Associated Homo eqn $f(n) - 3f(n-1) - 4f(n-2) = 0$

For char eqn, take $f(n) = a^n$

$$\Rightarrow a^2 - 3a - 4 = 0$$

$$\Rightarrow a = \underline{4}, -1$$

$$\therefore f^h(n) = A(4)^n + B(-1)^n$$

Particular soln

As $g(n) = 4^n$ where 4 is char root.

Take $f(n) = n\underline{d(4)^n}$ as particular soln in the given recc. relation

$$f(n) - 3f(n-1) - 4f(n-2) = 4^n$$

$$\Rightarrow nd(4)^n - 3(n-1)d(4)^{n-1} - 4(n-2)d(4)^{n-2} = 4^n$$

$$\Rightarrow nd(4)^2 - 3(n-1)d(4) - 4(n-2)d = 4^2$$

$$\Rightarrow 16nd - 12(n-1)d - 4(n-2)d = 16$$

$\Rightarrow \quad (16n - 12n + 12 - 4n + 8)\, d = 16$

$$\Rightarrow \quad 20d = 16$$

$$\Rightarrow \quad d = \frac{4}{5}$$

$\therefore \quad f^p(n) = nd\,(4)^n = n\left(\frac{4}{5}\right)(4)^n = \frac{n}{5}(4)^{n+1}$

Complete soln

$f(n) = f^h(n) + f^p(n)$

$= A(4)^n + B(-1)^n + \frac{n}{5}(4)^{n+1}$ Ans

If $q(n) = (q_0 + q_1 n + \ldots + q_k n^k)\, b^n$

For particular soln, $f(n) = n^m (d_0 + d_1 n + \ldots + d_k n^k)\, b^n$
where m is the multiplicity of b as char root.

$q_1 = q_2 \ldots = q_k = 0,$	$b = 1$	$q(n) = q_0$ const.
$q_2 = \ldots = q_k = 0$	$b = 1$	$q(n) = q_0 + q_1 n$ linear
$b = 1$		$q(n) = q_0 + q_1 n + \ldots + q_k n^k$ polyn of degree k
$q_1 = \ldots = q_k = 0$		$q(n) = q_0 b^n$

Q:→ Solve $f(n) + f(n-1) = 3n(2)^n$

Sol:→ Homo soln $f^h(n)$:

Associated Homo eqn $f(n) + f(n-1) = 0$

For char eqn, take $f(n) = a^n$

$$\Rightarrow a+1=0$$

$$\Rightarrow a=-1$$

$$\therefore f^h(n) = A(-1)^n$$

Particular soln $f^p(n)$:

$q(n) = 3n(2)^n$

For particular soln, take $f(n) = (d_0 + d_1 n)2^n$ in the given recc. relation

$$f(n) + f(n-1) = 3n(2)^n$$

$$\Rightarrow [d_0 + d_1 n]2^n + [d_0 + d_1(n-1)]2^{n-1} = 3n(2)^n$$

$$\Rightarrow (d_0 + d_1 n)2 + (d_0 + nd_1 - d_1) = 3n \cdot (2)$$

$$\Rightarrow 2d_0 + 2d_1 n + d_0 + nd_1 - d_1 = 6n$$

$$\Rightarrow 3d_0 - d_1 + 3d_1 n = 6n$$

Equate const term and coeff of n

$$3d_0 - d_1 = 0 \quad , \quad 3d_1 = 6$$

$$3d_0 - 2 = 0 \qquad d_1 = 2$$

$$d_0 = \frac{2}{3}$$

$$\therefore f^p(n) = (d_0 + d_1 n)2^n = \left(\frac{2}{3} + 2n\right)2^n$$

$$= \left(\frac{1}{3} + n\right)2^{n+1}$$

Complete soln.

$$f(n) = A(-1)^n + \left(\frac{1}{3} + n\right)2^{n+1}$$ Ans